Content Standards:

Data Analysis and Probability

**Benchmark A**
Create, interpret and use graphical displays and statistical measures to describe data; e.g., box-and-whisker plots, histograms, scatterplots, measures of center and variability.

**Indicator 1**
Use, create and interpret scatter plots and other types of graphs as appropriate.

**Benchmark F**
Construct convincing arguments based on analysis of data and interpretation of graphs.

**Indicator 6**
Make conjectures about possible relationship in a scatterplot and approximate line of best fit.

**Indicator 9**
Construct convincing arguments based on analysis of data and interpretation of graphs.

Number, Number Sense and Operations

**Benchmark G**
Estimate, compute, and solve problems involving real numbers, including ratio, proportion and percent, and explain solutions.

**Indicator 6**
Estimate, compute, and solve problems involving rational numbers, including ratio, proportion and percent, and judge the reasonableness of solutions.

Measurement

**Benchmark A**
Solve increasingly complex non-routine measurement problems and check reasonableness or results.

**Indicator 6**
Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.

**Benchmark F**
Write and solve real-world, multi-step problems involving money, elapsed time and temperature, and verify reasonableness of solutions.

**Indicator 6**
Solve and determine the reasonableness of the results for problems involving rates and derived measurements, such as velocity and density, using formulas, models and graphs.
Mathematical Processes Benchmarks

A. Formulate a problem or mathematical model in response to a specific need or situation, determines information required to solve the problem, to choose a method for obtaining this information, and to set limits for acceptable solution.
F. Use precise mathematical language and notations to represent problem situations and mathematical ideas.
G. Write clearly and coherently about mathematical thinking and ideas.
H. Locate and interpret mathematical information accurately, and communicate ideas, processes and solution in a complete and easily understood manner.

Pre-Assessment:
• Direct the students to complete Show What You Know, Attachment A, which contains exercises and problem situations for the following concepts:
  a. Ability to understand rate,
  b. Ability to plot points,
  c. Ability to understand features of a graph, (appropriate scales, variables on appropriate axis) and
  d. Ability to use appropriate and reasonable measures.
• After completing the pre-assessment activity, students choose partners and discuss their understanding of the activity. Facilitate a class discussion that leads to the correct answers for the pre-assessment exercises.

Scoring Guidelines:
Use observations and a checklist, or rubric to score the Pre-Assessment Activity. See Attachment B, Scoring Guidelines. Based on assessment results, additional practice in understanding rates, plotting points, features of a graph and using appropriate and reasonable measures may be needed.

Post-Assessment:
Have students complete a performance task.
• Design a real-world problem involving money, elapsed time or temperature and verify the reasonableness of solutions.
• Collect, organize, and represent data using tables and graphs.
• Provide mathematical evidence of understanding how to solve nonroutine measurement problems.
• Check reasonableness of results through either a brief presentation or a written paragraph.

Scoring Guidelines:
Use a variety of strategies to gather evidence of student understanding. Create rubrics or scoring guidelines as a class before beginning the project to outline the level of expectations for the students and clarify aspects of the assignment. To create a class rubric, ask students what items they believe should appear in the project to show mathematical understanding. Ask them to clarify each item into levels of performance.
Instructional Procedures:
Part One
1. Complete the pre-assessment.
2. Ask students to predict how long it will take to walk the length of the school hallway and how long they believe the hallway is. Also, have them estimate the length of their stride (stride length). Record these individual predictions on Attachment C, *How Far Can You Walk? Data Collection Sheet*.
3. Divide students into groups of no more than four and assign each student a specific task: walker, timer, measurement specialist and recorder.

Instructional Tip:
Each student acts as walker, timer, measurement specialist, and recorder. Students rotate jobs throughout the data collection in this activity. Place a chart on the board outlining the order in which they perform each task:

Walker —> Timer —> Measurement Specialist —> Recorder —> Walker

4. Have students discuss in groups the reasonableness of their predictions. To help students determine if their predictions are reasonable, brainstorm as a class the meaning of reasonable. Some suggestions may include:
   - Not reasonable: The bus ride to school took three hours. The walk to school can be done at a rate of 50 mph.
   - Reasonable: The width of a student desk is 18 inches. It will take two hours to travel 100 miles traveling at a speed of 50 mph.
5. Direct students to choose a unit of measure (meters, feet, yards, or kilometers). Then, each member walks the length of the hallway: The walker walks the length of the hallway counting steps taken while the timer records the time; the measurer measures the length of the hallway and recorder records all the data on Attachment C.

Instructional Tip:
Require students to measure and record the length of the hallway four times to reinforce the concepts of measurement and measures of center. Discuss measurement. Suggest the timer walk along side the walker to increase timing accuracy. All students must copy and share data on their own collection sheets.

6. Have students in the group rotate (or change roles) which will enable the four measurements.
7. Have groups calculate the walking rate using the distance formula:
   \[
   \text{Distance} = (\text{Rate}) \times (\text{Time})
   \]
8. Have groups calculate measures of center (mean, median and mode) as well as range, and list outliers for the collected data.
**Instructional Tip:**
Calculating measures of center should be a review for students and can prompt discussion of appropriate measures of central tendency.

9. Summarize the concepts in the lesson.
   a. Ask the students to reflect on the day’s activities.
   b. Ask the students to think of vocabulary words that relate to this lesson. List these on the board.
   c. Ask the students some questions to help them process the lesson.
   d. Suggested questions might include:
      - What is rate?
      - How does your walking rate compare to each of your other group members?
      - Why do you think we counted steps?
      - How do you know that your answer is reasonable?
      - How would you describe distance?
      - Describe the process of calculating distance.
      - Would you ever use this in the real world? If so when?
   e. Students complete *What I Know*, Attachment D, before leaving class.

**Part Two**

10. Ask students questions to develop their understanding of stride length. Formulate questions based on data from the prior day, *What I Know*, Attachment D. Suggested questions:
   - What is meant by stride length?
   - How is stride length determined?
   - Once the number of steps per unit of distance is determined, what purpose will that information serve?
   - In what form is the information most reasonable and why? (length of the hallway/steps or steps/length of the hallway, ft/steps or steps/ft or m/steps or steps/m)
   - How can knowing one’s stride length be beneficial?
   - How is knowing one’s stride length beneficial to the measurement of the length of the hallway?
   - Would stride increase, decrease or remain the same when you walk, jog or run?
   - Would you ever use this in the real world? If so when?
   - In what careers would you use this?

**Instructional Tip:**
Ask questions in a whole-class format or have students share with another student as partners, then two sets of partners will discuss their understanding before actually measuring stride length.

11. Have the students calculate and record their personal stride length on *How Far Can You Walk Data Sheet*, Attachment C, by completing the following steps:
   a. Measure a short distance in the classroom (for example: three meters, 10 feet, two yards).
   b. Each student walks the distance, counting his/her steps.
   c. Record the steps on *How Far Can You Walk Data Sheet*, Attachment C.
   d. Calculate individual stride length and record results
      Stride length: distance walked _____ Steps taken _____
For example: three meters, six steps:
\[
\frac{3\text{ meters}}{6\text{ steps}} \quad \text{is equal to one-half meter (.5) per step}
\]

Steps taken _____ \hspace{1cm} \text{Distance walked _____}

For example: 3 meters, 6 steps:
\[
\frac{6\text{ steps}}{3\text{ meters}} \quad \text{is equal to two steps per meter}
\]

12. Students calculate the distance of the hallway using their personal stride length and the number of steps it took them to walk the hallway.

For example: Student A walked three meters taking six steps and walked the hallway taking 220 steps. How long is the hallway?

\[
\frac{3\text{ meters}}{6\text{ steps}} = \frac{n\text{ meters}}{220\text{ steps}} \quad n = 110\text{ meters}
\]

13. Ask students: How does the calculated value of the hallway length compare to your individual and average measures of the hallway?

14. Have students predict how far they walk per day given their calculation of stride length and their experience with the activity.

15. Construct graphs of the data and describe what the graphs reveal.

Suggested items for graphs:

1. Rates of time
2. Stride length

For Example:

```
| Rate | Time |
```

“Time” represents each group member’s time to walk the hallway and “rate” represents each group member’s walking rate.

For Example:

```
| Time | Stride Length |
```

“Time” reflects each group member’s time to walk the hallway and “stride length” is each group member’s stride length.
**Instructional Tip:**
Students determine appropriate data to graph and interpret their graphs in a manner that demonstrates mathematical understanding. Discuss slope as a rate of change at this point in the lesson.

16. Ask students to discuss graphs within their groups and then share with the class what they learned from the activity. Ask students some reflective questions:
   - What information is most helpful on a graph? Why?
   - What type of graph best displays the data? Why?
   - How can knowing one’s stride length be beneficial to the measurement of a set distance?
   - What is meant by stride length?
   - How is stride length determined?
   - Would you ever use this in the real world? If so, where?
   - Can you determine how many steps it would take to walk a mile given the information you have collected? Explain.

17. Summarize the concepts of the lesson.
   a. Review mathematical terms from the lesson, discuss and clarify understanding and have students write definitions in their own words.
   b. Summarize the big ideas of the lesson and have students reflect on their learning using guiding questions.
      - What did you learn?
      - Is it reasonable that an average male can walk 1,000,000 steps a day? Why or why not?
      - Is it reasonable that an average female walks four miles per day? Justify your rationale.
      - How many steps would it take to walk a mile? Support the rationale.
      - How does stride affect distance walked over a set time?
      - How does stride affect the number of steps taken over a set distance?
      - When would it be best to use yards per minutes instead of per feet per minutes? (Or meters and kilometers)
      - How can you calculate the cost per mile if you travel by plane or by car? Assume it costs $.36 per mile to drive.
      - If you start school at 8:15 a.m. and you have to walk two miles to school one way, what time do you have to leave your house? Support your answer.

**Instructional Tip:**
Inspire student reflection using questions. Divide the students into four groups and locate each in a corner of the room.
   a. Each student reports to the assigned corner based on the last task held. Students should be directed to a corner of the room as follows:
      - Walkers report to corner one of the room.
      - Timers report to corner two of the room.
      - Measurement specialists report to corner three of the room.
      - Recorders report to corner four of the room.
   b. Assign one reflection question for the students in each group to discuss.
   c. They display their responses in chart form and then report their findings to the class.
d. Address and clarify misconceptions and reinforce the lesson.

**Differentiated Instructional Support:**
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s). Using the knowledge domain of revised Bloom’s taxonomy can assist the development of differentiated standards and assessments.

- All students make estimates and collect data.
- All students grow in understanding and application of the vocabulary: rate, stride length, distance and reasonableness.
- Students who require additional assistance benefit from teacher-directed instruction to complete the activity (assistance in calculating stride length, developing graphs, and interpreting results, designing a problem for the Post Assessment) and additional activities to apply their knowledge and skill.
- High-achieving students devise other uses for this data and make other conjectures supported by research (for example, distance walked for individuals living in the city vs. the country). Encourage them to design and then evaluate more complicated real-world problems involving more than one multi-step problem (e.g., problems that measure distance, rate, time and interest).
- A chart and the attachments help visual learners organize data. Working with others and responding to teacher and student questions benefits auditory learners. Kinesthetic learners enjoy the physical movement of this activity.
- Instructional strategies range in complexity from recalling basic knowledge on rates and measurements to actual application, analysis, creation and evaluation.
- Teacher observation, the pre-assessment, the instruction and the post-assessment provide assessment for both formative and summative evaluation. Make appropriate choices by varying degree of complexity of problems from simple to complex.
- Technology connections help reach students in need of intervention and provide extensions for high-achieving students.
- Students in need of intervention benefit from additional guidance as they begin the activity. Remind them how to read a ruler, use a timing device and repeat instructional procedures, as necessary.

- Require accelerated students to devise additional questions to explore the data. Encourage them to create a variety of graphical displays and evaluate their graphs. Students find examples of rates that appear in newspapers, magazines and other media. Ask them to invent problems that involve more than one step (e.g., problems that involve temperature, interest and distance, rate, time). Encourage students at this level to use a motion detector (Calculator-Based Ranger), graphing calculator and/or computer spreadsheets and graphs.

**Extensions:**
- Students walk the hallway wearing pedometers. Compare the steps recorded on the pedometers to those recorded during the activity. Explore walking distances and steps taken with the pedometers.
- Enter data into a spreadsheet and design graphs using the data.
- Students design and complete a race car activity (small model cars) in which they record distance and time and work to find the rate at which the car travels. Students build a race track where their model cars will run. They place their cars on the track and record the distance it travels and the time it is in motion. Students may use a fixed incline or may find it interesting to explore what happens when different inclines are used. Include a graph of the data collected and a reflection of the activity as part of this extension.
- Students design a trip for which they research hotel rates, explore travel by car, plane and train and determine the cost of the trip during a selected period of time. Include a discussion of how the expenses will be paid, (by cash, check or credit card) and how long it will take to save for the trip and/or pay off the trip given the method of payment.
- Explore what it looks like to walk the hallway or another set distance using a Calculator-Based Ranger (CBR).

**Materials and Resources:**

*For the teacher:* Measuring tapes, rulers, trundle wheel (optional), stop watches, chart paper, graph paper, calculators and other technology as appropriate

*For the student:* Calculator

**Technology Connections:**

- Possible technology tools and materials which will enhance learning in this lesson include pedometer, calculator, graphing calculator, spreadsheet, calculator-based ranger (CBR) and the Internet.

**General Tips:**

- This is a two-day lesson. Be flexible. Depend on the students’ ability to calculate, ability to set up graphs and work effectively in groups, the lesson may take longer.

**Attachments:**

Attachment A, *Show What You Know!*
Attachment B, *Show What You Know! Scoring Guidelines*
Attachment C, *How Far Can You Walk? Data Collection Sheet*
Attachment D, *What I Know*

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**Attachment A**
Show What You Know!

Name: ____________________________  Date:________________________

**Derive Rates:**

1. How do you determine the cost per pencil if a dozen cost $2.28?

2. a) Using the table below determine the number of books read in 1 hour.

   b) Using the table below determine the number of hours required to read 10 books.

<table>
<thead>
<tr>
<th>Reading time</th>
<th>Books read</th>
</tr>
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<tbody>
<tr>
<td>5 hours</td>
<td>2</td>
</tr>
<tr>
<td>7.5 hours</td>
<td>3</td>
</tr>
<tr>
<td>10 hours</td>
<td>4</td>
</tr>
<tr>
<td>12.5 hours</td>
<td>5</td>
</tr>
</tbody>
</table>

**Plot points:**

1. Label each axis.

2. Identify the coordinates of each point:

   Point A ______________
   Point B ______________
   Point C ______________
   Point D ______________
3. Plot each point on the coordinate graph.

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>(-4,3)</td>
</tr>
<tr>
<td>I</td>
<td>(0,5)</td>
</tr>
<tr>
<td>J</td>
<td>(-3,-8)</td>
</tr>
<tr>
<td>K</td>
<td>(7,8)</td>
</tr>
</tbody>
</table>
Directions: Examine the following graphs; determine which graph is the most reasonable for this situation: Circle the most reasonable answer.

Determine Reasonable Measures
1. Is it reasonable that a car can travel 400 miles in 2 hours? Why or why not?

2. Is it reasonable to assume that a diamond with a mass of 4.25 g and a volume of 3 cm$^3$, has a density of 7 g/cm$^3$? (Where $d=m/v$) Why or why not?
## Attachment B
### Pre-Assessment Scoring Options

**Checklist**

<table>
<thead>
<tr>
<th>Name:</th>
<th>Complete Understanding</th>
<th>Partial Understanding</th>
<th>Limited Understanding</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Accurate computation, appropriate procedure</td>
<td>Computation error or inappropriate procedure</td>
<td>No attempt, inaccurate computation, or inappropriate procedure</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Task</th>
<th>Complete Understanding</th>
<th>Partial Understanding</th>
<th>Limited Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derive rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plot points</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Match graph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine reasonable measures</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Attachment C
How Far Can You Walk? Data Collection Sheet

Name _______________________________________

Group Members _______________________________________________________

Before working with your group complete items 1-3:

1. Estimated time to walk the school hallway ____________________________

2. Estimated length of the hallway _________________________________

3. Estimated stride length _________________________________________

As a group:

4. Decide the unit of measure that you will use to measure the length of the hallway. ______________

5. With your group, walk the hallway, record the data:

<table>
<thead>
<tr>
<th>Walker</th>
<th>Time to walk hallway</th>
<th>Hallway length (distance)</th>
<th>Number of steps taken while walking the hallway</th>
<th>Walking Rate Distance (Rate)(Time)</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

*Mean*

*Median*

*Mode*

*Range*

*List any Outliers*
Attachment C (Continued)
How Far Can You Walk? Data Collection Sheet

*Complete Individually:*

6. Calculate personal stride length:

   a. *Distance walked*  
      Steps taken

   b. *Steps taken*  
      Distance walked

7. What does each calculation in #6 tell us? Explain fully.

8. Calculate the length of the hallway using your personal stride length and the number of steps taken to walk the hallway. (Hint: set up a proportion.)

9. How does the length you found in #8 compare to what was measured by each group member? How does the length you found compare to the group average length?

10. What would account for any differences in length? Explain fully.

11. Using the unit measure your group chose and applying your personal stride length, how far do you walk per day? Explain your rationale.
13. Compare your original estimates to the actual data collected. Reflect on what this comparison shows.

14. Would you estimate differently next time? Why or why not?
## Attachment D
### What I Know…

<table>
<thead>
<tr>
<th>Mathematical Terms</th>
<th>What I Know</th>
<th>I Still Need To Know</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stride Length</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reasonableness</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>